ACTIVITY 4.1

Exponent Rules

Activity Focus
- Exponent rules
- Scientific notation
- Meaning of negative exponents
- Meaning of a number to a zero power

Materials
- No special materials are needed.

Chunking the Activity
#1 #11–12 #21
#2–5 #13–14 #22
#6–7 #15–16 Try These E
#8–9 #17–18
#10 #19–20

First Paragraph Marking the Text
Students will use the analysis of icebergs to develop the ideas of exponent rules, negative exponents and scientific notation. Some relationships have been simplified to make calculations easier.

Icebergs and Exponents

SUGGESTED LEARNING STRATEGIES: Marking the Text, Group Discussion, Create Representations, Predict and Confirm

An iceberg is a large piece of freshwater ice that has broken off from a glacier or ice shelf and is floating in open sea water. Icebergs are classified by size. The smallest sized iceberg is called a “growler”.

A growler was found floating in the ocean just off the shore of Greenland. Its volume above water was approximately 27 cubic meters.

1. Two icebergs float near this growler. One iceberg’s volume is $3^4$ times greater than the growler. The second iceberg’s volume is $2^8$ times greater than the growler. Which iceberg has the larger volume? Explain below.

2. What is the meaning of $3^4$ and $2^8$? Why do you think exponents are used when writing numbers?

3. Suppose the original growler’s volume under the water is 9 times the volume above. How much of its ice is below the surface?

4. Write your solution to Item 3 using powers. Complete the equation below. Write the missing terms as a power of 3.

   $$\text{volume above water} \cdot 3^2 = \text{volume below the surface}$$

5. Look at the equation you completed for Item 4. What relationship do you notice between the exponents on the left side of the equation and the exponent on the right?

   The exponent on the right side of the equation is the sum of the exponents on the left.

CONNECT TO GEOLOGY
Because ice is not as dense as sea water, about one-tenth of the volume of an iceberg is visible above water. It is difficult to tell what an iceberg looks like underwater simply by looking at the visible part. Growlers got their name because the sound they make when they are melting sounds like a growling animal.
**ACTIVITY 4.1 Continued**

6. **Look for a Pattern**
Throughout this activity, students complete tables to illustrate patterns with exponents. After each table question, students generalize their findings in the next question.

7. **Predict and Confirm, Think/Pair/Share, Create Representations, Debriefing, Note Taking**
Students should be given the opportunity to share their thoughts on what the missing exponent should be, but a debriefing should clarify the property as $a^m \cdot a^n = a^{m+n}$. Students should write the property in their notebooks.

8. **Predict and Confirm**
It is expected that students will use a calculator on this problem. It is understood that some teachers prefer to have students do computations by hand to improve their numeracy skills. However, the intent of this lesson is to have students explore exponents, and computing by hand can distract from the intent of the lesson and slow students’ progress on this topic.

9. **Think/Pair/Share, Debriefing, Note Taking**
Students should be given the opportunity to use guess and test to help them to change the numbers in Item 8 to powers of 9. They will discover that $59,049 = 9^5$ and $729 = 9^3$. Thus, $9^5 \div 9^2 = 9^3$.

10. **Group Discussion**
This question asks students to once again predict a pattern based on one example. Some students may know or can predict the Quotient of Powers rule for exponents. A similar strategy was used for Item 5. Have students share answers on their patterns. They will then use the following questions to verify their predictions of the rule.

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**SUGGESTED LEARNING STRATEGIES:** Look for a Pattern, Predict and Confirm, Think/Pair/Share, Create Representations, Note Taking, Group Discussion

6. Use the table below to help verify the pattern you noticed in Item 5. First write each product in the table in expanded form. Then express the product as a single power of the given base. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Product</th>
<th>Expanded Form</th>
<th>Single Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^2 \cdot 2^3$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>$2^5$</td>
</tr>
<tr>
<td>$3^4 \cdot 3^2$</td>
<td>$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$</td>
<td>$3^6$</td>
</tr>
<tr>
<td>$a^m \cdot a^n$</td>
<td>$a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$</td>
<td>$a^{m+n}$</td>
</tr>
</tbody>
</table>

7. Based on the pattern you observed in the table in Item 6, write the missing exponent in the box below to complete the **Product of Powers Property** for exponents.

$$a^m \cdot a^n = a^{m+n}$$

8. The density of an iceberg is determined by dividing its mass by its volume. Suppose a growler had a mass of 59,049 kg and a volume of 81 cubic meters. Compute the density of the iceberg. $729$ kg/m³

9. Write your solution to Item 8 using powers of 9.

$$\frac{\text{Mass}}{\text{Volume}} = \text{Density}$$

$$\frac{9^5}{9^1} = 9^4$$

10. What pattern do you notice in the equation you completed for Item 9?
The exponent on the right side of the equation is the difference between the exponents on the left.
11. Use the table to help verify the patterns you noticed in Item 9. First write each quotient in the table below in expanded form. Then express the quotient as a single power of the given base. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Product</th>
<th>Expanded Form</th>
<th>Single Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2^5}{2^2} )</td>
<td>( \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} )</td>
<td>( 2^3 )</td>
</tr>
<tr>
<td>( \frac{5^6}{5^5} )</td>
<td>( \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} )</td>
<td>( 5 )</td>
</tr>
<tr>
<td>( \frac{a^3}{a} )</td>
<td>( \frac{a \cdot a \cdot a}{a} )</td>
<td>( a^2 )</td>
</tr>
<tr>
<td>( \frac{x^7}{x^3} )</td>
<td>( \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} )</td>
<td>( x^4 )</td>
</tr>
</tbody>
</table>

12. Based on the pattern you observed in Item 11, write the missing exponent in the box below to complete the Quotient of Powers Property for exponents.

\[ \frac{a^n}{a^m} = a^{m-n} \]

The product and quotient properties of exponents can be used to simplify expressions.

**EXAMPLE 1**

Simplify: \( 2x^5 \cdot 5x^3 \)

**Step 1:** Group powers with the same base. \( 2x^5 \cdot 5x^3 = 2 \cdot 5 \cdot x^5 \cdot x^3 \)

**Step 2:** Product of Powers Property \( = 10x^{5+3} \)

**Step 3:** Simplify the exponent. \( = 10x^8 \)

**Solution:** \( 2x^5 \cdot 5x^3 = 10x^8 \)
**EXAMPLE 2** Note Taking

**TRY THESE A** These items will provide a valuable tool to determine understanding of the rules in a more complex set-up. Students are not expected to have necessarily mastered these specific questions at this point. These items should be used as a formative assessment, and can aid instructors by having groups share answers and process on whiteboards. Students that need more support and help can be identified and any interventions needed to support their learning can be implemented.

**13-16 Activating Prior Knowledge, Predict and Confirm** Students will take what they have learned about the quotient property and extend it to negative exponents and a base to the zero power.

**EXAMPLE 2**

Simplify \( \frac{2x^3y^4}{xy^2} \).

**Step 1:** Group powers with the same base.

\[
\frac{2x^3y^4}{xy^2} = \frac{2 \cdot x^3 \cdot y^4}{x \cdot y^2}
\]

**Step 2:** Quotient of Powers Property

\[
= 2x^{3-1} \cdot y^{4-2}
\]

**Step 3:** Simplify the exponent.

\[
= 2x^2
\]

**Solution:**

\[
\frac{2x^3y^4}{xy^2} = 2x^2
\]

**TRY THESE A**

Simplify each expression.

\[
a. \quad (4xy^3)(-2x^2y^5) \\
\]

**b.** \( \frac{2a^6b^3}{4ab^3} \)

**c.** \( \frac{6y^4 \cdot 2xy}{3} \)

13. Write each quotient in expanded form and simplify it. Then apply the quotient property of exponents. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Quotient</th>
<th>Expanded Form</th>
<th>Single Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2^3}{2^2} )</td>
<td>( \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} )</td>
<td>( 2^{3-2} = 2 )</td>
</tr>
<tr>
<td>( \frac{a^4}{a^3} )</td>
<td>( a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a )</td>
<td>( a^{1-0} = a )</td>
</tr>
<tr>
<td>( \frac{x^6}{x^3} )</td>
<td>( x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x )</td>
<td>( x^{4-0} = x^4 )</td>
</tr>
</tbody>
</table>
14. Based on the pattern you observed in Item 13, write the missing exponent in the box below to complete the **Negative Power Property** for exponents.

\[
\frac{1}{a^n} = a^{-n}
\]

15. Write each quotient in expanded form and simplify it. Then apply the quotient property of exponents. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Quotient</th>
<th>Expanded Form</th>
<th>Single Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2^4}{2^3} )</td>
<td>( \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} )</td>
<td>( 2^{4-3} = 2^1 )</td>
</tr>
<tr>
<td>( \frac{5^6}{5^4} )</td>
<td>( \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5} )</td>
<td>( 5^{6-4} = 5^2 )</td>
</tr>
<tr>
<td>( \frac{a^3}{a^2} )</td>
<td>( \frac{a \cdot a \cdot a}{a \cdot a} )</td>
<td>( a^{3-2} = a^1 )</td>
</tr>
</tbody>
</table>

16. Based on the pattern you observed in Item 15, fill in the box below to complete the **Zero Power Property** of exponents.

\[ a^0 = 1 \]

You can use the negative power property and the zero power property of exponents to evaluate and simplify expressions.

**TRY THESE B**

Simplify each expression.

a. \( 2^{-3} \cdot \frac{1}{8} \)  

b. \( \frac{10^2}{10^{-3}} \)  

c. \( 3^{-1} \cdot 5^1 \)  

D. \( (-3.75)^0 \)
When evaluating and simplifying expressions, you can apply the properties of exponents and then write the answer without negative or zero powers.

**EXAMPLE 3**

Simplify \(5x^{-2}y^2 \cdot \frac{3x^4}{y^3}\) and write without negative powers.

**Step 1:** Commutative Property

\[
5x^{-2}y^2 \cdot \frac{3x^4}{y^3} = 5 \cdot 3 \cdot x^{-2+4} \cdot y^{2-3+1} \cdot z^0
\]

**Step 2:** Apply the exponent rules.

\[
5 \cdot 3 \cdot x^{2} \cdot y^{-1} \cdot z^0
\]

**Step 3:** Simplify the exponents.

\[
= 15 \cdot x^{2} \cdot y^{-1} \cdot 1
\]

**Step 4:** Write without negative exponents.

\[
= \frac{15x^2}{y}
\]

**TRY THESE C**

Simplify and write without negative powers.

a. \(2ab^{-3} \cdot 5ab\)  

b. \(10x^3 \cdot y^{-4} \cdot \frac{2x^4}{y^2}\)  

c. \((-3xy^{-3})^{4}\)

17. Write each expression in expanded form. Then write the expression using a single exponent with the given base. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Expression</th>
<th>Expanded Form</th>
<th>Single Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2^3)^4)</td>
<td>(2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2)</td>
<td>(2^{12})</td>
</tr>
<tr>
<td>((5^2)^3)</td>
<td>(5^6 \cdot 5^6 \cdot 5^6)</td>
<td>(5^{12})</td>
</tr>
<tr>
<td>((x^3)^4)</td>
<td>(x^{12} + x^{12} + x^{12} + x^{12})</td>
<td>(x^{12})</td>
</tr>
</tbody>
</table>
18. Based on the pattern you observed in Item 17, write the missing exponent in the box below to complete the **Power of a Power Property** for exponents.

\[(a^n)^m = a^{nm}\]

19. Write each expression in expanded form and group like terms. Then write the expression as a product of powers. The first one has been done for you.

<table>
<thead>
<tr>
<th>Original Expression</th>
<th>Expanded Form</th>
<th>Product of Powers</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2x)^4)</td>
<td>(2x \cdot 2x \cdot 2x \cdot 2x = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x)</td>
<td>(2^4x^4)</td>
</tr>
<tr>
<td>((-4a)^3)</td>
<td>(-4a \cdot -4a \cdot -4a = -4 \cdot -4 \cdot -4 \cdot a \cdot a \cdot a)</td>
<td>((-4)^3a^3)</td>
</tr>
<tr>
<td>((x^2y^3)^5)</td>
<td>(x^2y^3 \cdot x^2y^3 \cdot x^2y^3 \cdot x^2y^3 = x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot y^3 \cdot y^3 \cdot y^3 \cdot y^3 \cdot y^3)</td>
<td>((x^2y^3)^5)</td>
</tr>
</tbody>
</table>

20. Based on the pattern you observed in Item 19, write the missing exponents in the boxes below to complete the **Power of a Product Property** for exponents.

\[(ab)^m = a^m \cdot b^m\]

21. Use the patterns you have seen. Predict and write the missing exponents in the boxes below to complete the **Power of a Quotient Property** for exponents.

\[\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\]
EXAMPLE 4 Note Taking, Activating Prior Knowledge

EXAMPLE 5 Note Taking,Activating Prior Knowledge

You can apply these power properties and the exponent rules you have already learned to simplify expressions.

EXAMPLE 4

Simplify \((2x^2y^3)(3x^3)^{-2}\) and write without negative powers.

Step 1: Power of a Power Property
\[
(2x^2y^3)(3x^3)^{-2} = 2x^2y^3 \cdot \frac{1}{3}x^{-6}
\]

Step 2: Simplify the exponents and the numerical terms.
\[
= 8 \cdot x^{\frac{4}{3}}y^{\frac{3}{3}} \cdot \frac{1}{3} \cdot x^{-4}
\]

Step 3: Commutative Property
\[
= 8 \cdot \frac{1}{3}x^{\frac{4}{3}} \cdot x^{-4}y^{\frac{3}{3}}
\]

Step 4: Product of Powers Property
\[
= \frac{8}{3}x^{\frac{4}{3} - 4}y^{\frac{3}{3}}
\]

Step 5: Simplify the exponents.
\[
= \frac{8}{3}x^{\frac{4}{3} - 4}y^{\frac{3}{3}}
\]

Solution:
\[
(2x^2y^3)(3x^3)^{-2} = \frac{8}{3}x^{\frac{4}{3}}y^{\frac{3}{3}}
\]

EXAMPLE 5

Simplify \(\left(\frac{x^2y^{-3}}{z}\right)^{\frac{1}{2}}\).

Step 1: Power of a Quotient Property
\[
\left(\frac{x^2y^{-3}}{z}\right)^{\frac{1}{2}} = \frac{x^2 \cdot y^{-3} \cdot z^{-\frac{1}{2}}}{z^{\frac{1}{2}}}
\]

Step 2: Simplify the exponents.
\[
= \frac{x^2}{z^{\frac{1}{2}}}
\]

Step 3: Negative Exponents Property
\[
= \frac{x^2}{y^3z}
\]

Solution:
\[
\left(\frac{x^2y^{-3}}{z}\right)^{\frac{1}{2}} = \frac{x^2}{y^3z}
\]
### TRY THESE D

Simplify and write without negative powers.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$(2x^2y^3)(-3xy)^2$</td>
</tr>
<tr>
<td>b.</td>
<td>$-2ab(5b^2c)^3$</td>
</tr>
<tr>
<td>c.</td>
<td>$\frac{(4x^3y^{-2})}{16x^2}$</td>
</tr>
<tr>
<td>d.</td>
<td>$\left(\frac{5x^3y}{10x^2}\right)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td>e.</td>
<td>$(3xy^{-2})(2x^3y^2)(6yz)^{-1}$</td>
</tr>
</tbody>
</table>

### EXAMPLE 6

Simplify $(1.3 \times 10^5)(4 \times 10^{-8})$.

**Step 1:** Group terms and use the Product of Powers Property.

$$\begin{align*}
(1.3 \times 10^5)(4 \times 10^{-8}) &= 1.3 \times 4 \times 10^{5-8} \\
&= 5.2 \times 10^{-3}
\end{align*}$$

**Step 2:** Multiply numbers and Simplify the exponent.

Solution: $(1.3 \times 10^5)(4 \times 10^{-8}) = 5.2 \times 10^{-3}$

### Writing Math

**Scientific notation** is used to express very large or very small numbers using powers of 10.

- $24,000,000 = 2.4 \times 10^7$
- $0.0000567 = 5.67 \times 10^{-5}$

Numbers written in scientific notation are always expressed as a product of a number between 1 and 10 and a power of 10.
ACTIVITY 4.1 Continued

CHECK YOUR UNDERSTANDING

1. \(x^{15}\)
2. \(2a^3b^6\)
3. \(-18a^7b^4\)
4. \(\frac{1}{2}xy\)
5. \(-\frac{1}{8}z^2\)
6. \(\frac{1}{6^2}\)
7. \(\frac{4}{x^2}\)
8. 1
9. \(\frac{16x^6}{y^3}\)
10. \(\frac{125x^3}{y^6}\)
11. \(-\frac{24a^4c^b}{b^2}\)
12. \(\frac{y^4}{-3x^3}\)
13. 12.5
14. \(3.155 \times 10^5\)
15. Answers may vary. Sample answer: When dividing numbers in scientific notation, divide the decimal parts by the decimal parts and the power-of-ten parts by the power-of-ten parts. The quotient may have to be adjusted so that it is in scientific notation.

TRY THESE

Express the product or quotient using scientific notation.

\(a. \quad (2.5 \times 10^{-3}) (1.5 \times 10^6)\)
\(b. \quad \frac{6.4 \times 10^{13}}{1.6 \times 10^5}\)
\(c. \quad \text{Compute the density of the iceberg described in Item 22.} \quad 9.201 \times 10^{24} \text{ kg/m}^3\)

WRITE YOUR ANSWERS ON NOTEBOOK PAPER.

Simplify and write each expression without negative exponents.

1. \(x^8 \cdot x^7\)
2. \(6a^3b^4\)
3. \((6a^2b)(-3ab^3)\)
4. \(7x^2y^3\)
5. \((-2x)^{-3}\)
6. \(\frac{b^6}{-2}\)
7. \(\frac{4x^{-2}}{x^3}\)
8. \((5x^{-y-2z})^0\)

9. \((4x^y)^{-1}\)
10. \(\frac{(5x^3)}{y^2}\)
11. \((-2a^b)(3ab^c)(xy2)\)
12. \(\frac{2xy^2}{x^3y^3} - \frac{5xy^3}{-3y^2}\)
13. \((2.5 \times 10^5)(5 \times 10^{-3})\)
14. \(\frac{6.31 \times 10^6}{2 \times 10^2}\)
15. MATHEMATICAL REFLECTION

What have you learned about simplifying expressions with exponents as a result of this activity?